Generic Programming with Type Families

— Scrap Your Container Instances —

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Motivation

Non-parametric container types

- A parametric container type (such as `[e]`)
  - uses a single representation
  - independent of the type of elements.

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  - in dependence on the type of elements it contains.
- This is usually for optimisation purposes.
- Examples of language features used for this purpose:
  - C++ (templates and traits)
  - Generic Haskell (type-indexed data types)
  - Haskell (type families)

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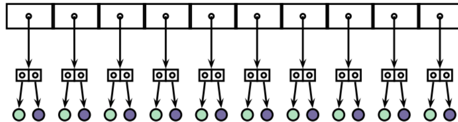
  - Haskell (type families) $\Longleftarrow$ This Talk!

In the development version of the Glasgow Haskell Compiler (GHC).

Boxed array:

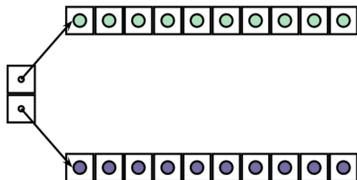


```
Array (Int, Bool)
```

Array example

```
data family Array e
```

Unboxed arrays:



```
ArrProd (ArrInt ua) (ArrBool bv) :: Array (Int, Bool)
```

Array example

```
data family Array e                 -- representation depends on e

data instance Array Int      = ArrInt  (UnbArray Int)
data instance Array Bool     = ArrBool BitVector
data instance Array (e1, e2) = ArrProd (Array e1)
                                       (Array e2)
```

Problem solved?!?

- Data familes obviously do the job
- Thanks for listening to this very short talk!

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The real problem solved in this talk

- Which type of elements can we store in such a non-parametric array?
- What about user-defined structures?
- Do we have to define a new set of array methods for every array element type?

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The goal

- Data instances for `Array` only for limited set of element types: basic types, products & sums, and arrays(!)
- The same for array method implementations
- Derive implementation for user-defined algebraic types automatically

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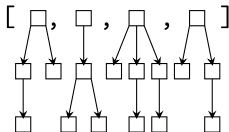
```
data MassPnt  -- mass points
data Tree     = Node MassPnt (Array Tree)
```



```haskell
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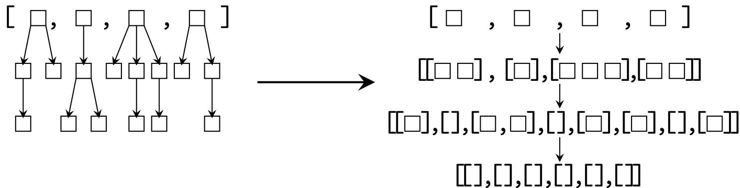

- Example application: Barnes-Hut algorithm to solve the *n*-body problem
- Tree performs spatial decomposition
- Levelwise representation ideal for parallel implementation

The Key Idea

> All problems in computer science can be solved by another level of indirection.
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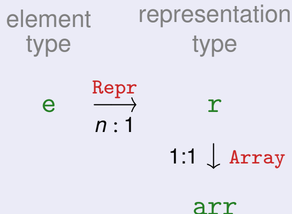
Two-level mapping



- `Repr`: type synonym family maps unbound number of algebraic data types to finite number of representations
- `Array`: data type family mapping element representations to array representations

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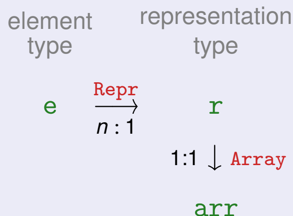

- `Repr`: type synonym family maps unbound number of algebraic data types to finite number of representations
- `Array`: data type family mapping element representations to array representations — We'll look at this point first.

Limited Set of Data Instances

What are our representation types going to be?

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```
data Int, ...                         -- basic types
data Unit     = Unit                  -- singleton
data a :*: b = a :*: b                -- products
data a :+: b = Inl a | Inr b          -- sums
```

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Type class categorising array elements

```
class ArrElem r where
  data Array r                 -- data family as associated type
  lengthA    :: Array r -> Int
  emptyA     :: Array r
  replicateA :: Int -> r -> Array r
  (!:)       :: Array r -> Int -> r
  -- and many more
```

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Class instances: basic types

```
instance ArrElem Int where
  data Array Int        = ArrInt (UnbArray Int)
  lengthA (ArrInt ua) = lengthUA ua
  emptyA                = ArrInt emptyUA
  replicateA n x        = ArrInt (replicateUA n x)
  (ArrInt a) !: i       = a!i
  – and so on
```

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data family Array r                     – arrays
```

Class instances: products

```
instance (ArrElem r1, ArrElem r2) =>
    ArrElem (r1 :*: r2) where
  data Array (r1 :*: r2) = ArrProd (Array r1) (Array r2)
  lengthA (ArrProd arr1 _) = lengthA arr1
  emptyA                   = ArrProd emptyPA emptyPA
  replicateA n (x1 :*: x2) = ArrProd (replicatePA n x1)
                                     (replicatePA n x2)
  (ArrProd a1 a2) !: i     = (a1!:i) :*: (a2!:i)
```

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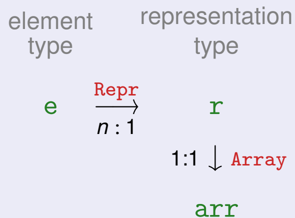
Class instances: and so on

```
instance ArrElem Unit where ...
instance (ArrElem r1, ArrElem r2) =>
    ArrElem (r1 :+: r2) where ...
instance ArrElem r => ArrElem (Array r) where ...
```

Remember…

Two-level mapping



Generic Representation Types

An embedding projection

```
class Representable a where
  type Repr a             – type function from a to representation of a
  toRepr   :: a -> Repr a            – embed
  fromRepr :: Repr a -> a            – project

  type Repr a = a         – default representation is the identity
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Default representations

```
instance Representable Int   -- and so on
instance Representable Unit
instance (Representable a, Representable b) =>
  Representable (a :*: b)
instance (Representable a, Representable b) =>
  Representable (a :+: b)
instance Representable e => Representable (PArray e)
```

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Example: spatial decomposition trees

```
data Tree = Node MassPnt (Array Tree)

instance Representable Tree where
  type Repr Tree = MassPnt :*: Array Tree
  toRepr   (Tree mpnt ts) = mpnt :*: toRepr ts
  fromRepr (mpnt :*: ts)  = Tree mpnt (fromRepr ts)
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```

Stub instance

```
instance ArrElem Tree where
  newtype Array Tree = ArrTree (Array (Repr Tree))
  lengthA (ArrTree a) = lengthA a
  emptyA              = ArrTree emptyA
  replicateA n e      = ArrTree (replicateA n (toRepr e))
  (ArrTree a) !: i    = fromRepr (a!:i)
```

Beyond the Basic Idea

Other choice of representation types possible

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Eliminate the stub instances

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- See `PArray` of Data Parallel Haskell

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Compiler support: deriving `Representable`

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Compiler support: deriving `Representable`

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data Tree = Node MassPnt (Array Tree)
          deriving Representable
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Conclusions

Non-parametric containers

- Goal: Avoid proliferation of container instances
- Key ideas:
  1. map user-defined types to fixed set of representation types
  2. type synonym family makes the mapping transparent
- A little compiler support makes it even more convenient

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Related work

- C++: unclear how to prevent tedious definition of instances for user-defined types
- Generic Haskell: special language extension and compiler; fixed set of representation types

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A Complete Example: Generalised Generic Tries

```
{-# LANGUAGE TypeFamilies, TypeOperators #-}
module GMap where

import Prelude hiding (lookup)
import Char (ord)
import qualified Data.Map as Map


-- Product-sum type representations
-- --------------------------------

data Unit = Unit
  deriving Show
data (:+:) a b = Inl a | Inr b
  deriving Show
data (:*:) a b = a :*: b
  deriving Show

class Representable a where
  type Repr a
  toRepr   :: a -> Repr a
  fromRepr :: Repr a -> a

instance Representable Unit where
  type Repr Unit  = Unit
  toRepr   = id
  fromRepr = id

instance Representable () where
  type Repr ()  = Unit
  toRepr ()     = Unit
  fromRepr Unit = ()
```

```
instance Representable Int where
  type Repr Int = Int
  toRepr   = id
  fromRepr = id

instance Representable Char where
  type Repr Char = Char
  toRepr   = id
  fromRepr = id

instance (Representable a, Representable b) =>
    Representable (a :*: b) where
  type Repr (a :*: b) = a :*: b
  toRepr   = id
  fromRepr = id

instance (Representable a, Representable b) =>
    Representable (a, b) where
  type Repr (a, b) = a :*: b
  toRepr (x, y)       = x :*: y
  fromRepr (x :*: y) = (x, y)

instance (Representable a, Representable b) =>
    Representable (a :+: b) where
  type Repr (a :+: b) = a :+: b
  toRepr   = id
  fromRepr = id
instance (Representable a, Representable b) =>
    Representable (Either a b) where
  type Repr (Either a b) = a :+: b
  toRepr (Left x)  = Inl x
  toRepr (Right y) = Inr y
  fromRepr (Inl x) = Left x
  fromRepr (Inr x) = Right x
```

```
-- Generalised maps
-- ----------------

class Representable k => GMapKey k where
  data GMap k :: * -> *
  empty       :: GMap k v
  lookup      :: k -> GMap k v -> Maybe v
  insert      :: k -> v -> GMap k v -> GMap k v

instance GMapKey Int where
  data GMap Int v = GMapInt (Map.Map Int v)
  empty           = GMapInt Map.empty
  lookup k (GMapInt m)
    = Map.lookup k m
  insert k v (GMapInt m)
    = GMapInt (Map.insert k v m)

instance GMapKey Char where
  data GMap Char v = GMapChar (GMap Int v)
  empty            = GMapChar empty
  lookup k (GMapChar m)
    = lookup (ord k) m
  insert k v (GMapChar m)
    = GMapChar (insert (ord k) v m)

instance GMapKey Unit where
  data GMap Unit v = GMapUnit (Maybe v)
  empty            = GMapUnit Nothing
  lookup Unit (GMapUnit v)
    = v
  insert Unit v (GMapUnit _)
    = GMapUnit $ Just v
```

```
instance (GMapKey a, GMapKey b) =>
    GMapKey (a :*: b) where
  data GMap (a :*: b) v = GMapPair (GMap a (GMap b v))
  empty          = GMapPair empty
  lookup (a :*: b) (GMapPair gm)
    = lookup a gm >>= lookup b
  insert (a :*: b) v (GMapPair gm)
    = GMapPair $ case lookup a gm of
        Nothing  -> insert a (insert b v empty) gm
Just gm2 -> insert a (insert b v gm2  ) gm

instance (GMapKey a, GMapKey b) =>
    GMapKey (a :+: b) where
  data GMap (a :+: b) v = GMapEither (GMap a v) (GMap b v
  empty                 = GMapEither empty empty
  lookup (Inl  a) (GMapEither gm1  _gm2)
    = lookup a gm1
  lookup (Inr b) (GMapEither _gm1 gm2 )
    = lookup b gm2
  insert (Inl  a) v (GMapEither gm1 gm2)
    = GMapEither (insert a v gm1) gm2
  insert (Inr a) v (GMapEither gm1 gm2)
    = GMapEither gm1 (insert a v gm2)
```

```
-- Derived instances
-- -----------------

-- Generic list representation
--
-- * Could be added to Typeable
-- * Could use n-ary sums and products
--
instance Representable [a] where
  type Repr [a] = Unit :+: (a :*: [a])
  toRepr []     = Inl Unit
  toRepr (x:xs) = Inr (x :*: xs)
  fromRepr (Inl Unit)        = []
  fromRepr (Inr (x :*: xs)) = x:xs

-- List-indexed maps
--
-- * Could also be generated
--
instance GMapKey a => GMapKey [a] where
  newtype GMap [a] v = GMapList (GMap (Repr [a]) v)
  empty                      = GMapList empty
  lookup k (GMapList gm)     = lookup (toRepr k) gm
  insert k v (GMapList gm) = GMapList $ insert (toRepr k) v gm
```